

$$1. \quad y''(t) + ty'(t) - 3y(t) = -t^2$$

$$y(0) = 3$$

$$y'(0) = -6$$

$$\Downarrow$$

$$y''(t) = -ty'(t) + 3y(t) + t^2$$

$$\begin{cases} x_1(t) = y(t) \\ x_2(t) = y'(t) \end{cases}$$

$$x_1'(t) = x_2(t)$$

$$x_2'(t) = -tx_2(t) - 3x_1(t) + t^2$$

$$x_1(0) = 3, x_2(0) = -6$$

$$2. \quad y''(t) = \cos(t-y) + y^2(t)$$

$$y(0) = 1$$

$$y'(0) = 0$$

$$\begin{cases} x_1(t) = y(t) \\ x_2(t) = y'(t) \end{cases}$$

$$\begin{cases} x_1'(t) = x_2(t) \\ x_2'(t) = \cos(t - x_1(t)) + x_1^2(t) \end{cases}$$

$$x_1(0) = 1, x_2(0) = 0$$

$$3. \quad y^{(4)}(t) - y^{(3)}(t) + 7y(t) = \cos t$$

$$y(0) = y'(0) = 1$$

$$y''(0) = 0$$

$$y^{(3)}(0) = 2$$

$$\Downarrow$$

$$y^{(4)}(t) = y^{(3)}(t) - 7y(t) + \cos t$$

$$\begin{cases} x_1(t) = y(t) \\ x_2(t) = y'(t) \\ x_3(t) = y''(t) \\ x_4(t) = y^{(3)}(t) \end{cases}$$

$$x_1'(t) = x_2(t)$$

$$x_2'(t) = x_3(t)$$

$$x_3'(t) = x_4(t)$$

$$x_4'(t) = x_4(t) - 7x_1(t) + \cos t$$

$$x_1(0) = x_2(0) = 1, x_3(0) = 0, x_4(0) = 2$$

$$4. \quad y^{(6)}(t) = [y'(t)]^3 - \sin(y(t)) + e^{2t}$$

$$y(0) = y'(0) = \dots = y^{(5)}(0) = 0$$

$$\begin{cases} x_1(t) = y(t) \\ x_2(t) = y'(t) \\ x_3(t) = y''(t) \\ x_4(t) = y^{(3)}(t) \\ x_5(t) = y^{(4)}(t) \\ x_6(t) = y^{(5)}(t) \end{cases}$$

$$x_1'(t) = x_2(t)$$

$$x_2'(t) = x_3(t)$$

$$x_3'(t) = x_4(t)$$

$$x_4'(t) = x_5(t)$$

$$x_5'(t) = x_6(t)$$

$$x_6'(t) = (x_2(t))^3 - \sin(x_1(t)) + e^{2t}$$

$$x_1(0) = x_2(0) = \dots = x_6(0) = 0$$

$$5. \quad x'' + y - x' = 2t \Rightarrow x'' = x' - y + 2t$$

$$x(3) = 5, \quad x'(3) = 2$$

$$y'' - x + y = -1 \Rightarrow y'' = x - y - 1$$

$$y(3) = 1, \quad y'(3) = -1$$

$$\begin{array}{l} x_1(t) = x \\ x_2(t) = x' \\ x_3(t) = y \\ x_4(t) = y' \end{array} \quad \left| \right.$$

$$x_1'(t) = x_2(t)$$

$$x_2'(t) = x_2(t) - x_3(t) + 2t$$

$$x_3'(t) = x_4(t)$$

$$x_4'(t) = x_1(t) - x_3(t) - 1$$

$$x_1(3) = 5, \quad x_2(3) = 2, \quad x_3(3) = 1, \quad x_4(3) = -1$$

$$6. \quad 3x'' + 5x - 2y = 0 \Rightarrow x'' = \frac{1}{3}(2y + 5x)$$

$$x(0) = -1, \quad x'(0) = 0$$

$$4y'' + 2y - 6x = 0 \Rightarrow y'' = \frac{1}{4}(6x - 2y)$$

$$y(0) = 1, \quad y'(0) = 2$$

$$\begin{array}{l} x_1(t) = x \\ x_2(t) = x' \\ x_3(t) = y \\ x_4(t) = y' \end{array} \quad \left| \right.$$

$$x_1'(t) = x_2(t)$$

$$x_2'(t) = \frac{1}{3}(2x_3(t) + 5x_1(t))$$

$$x_3'(t) = x_4(t)$$

$$x_4'(t) = \frac{1}{4}(6x_1(t) - 2x_3(t)) = \frac{1}{2}(3x_1(t) - x_3(t))$$

$$x_1(0) = -1, \quad x_2(0) = 0, \quad x_3(0) = 1, \quad x_4(0) = 2$$

$$7. \quad x''' - y = t \Rightarrow x''' = y + t$$

$$x(0) = x'(0) = x''(0) = 4$$

$$2x'' + 5y'' - 2y = 1 \Rightarrow y'' = \frac{1}{5}(2y - 2x'' + 1)$$

$$y(0) = y'(0) = 1$$

$$\begin{array}{l} x_1(t) = x \\ x_2(t) = x' \\ x_3(t) = x'' \\ x_4(t) = y \\ x_5(t) = y' \end{array} \quad \left| \right.$$

$$x_1'(t) = x_2(t)$$

$$x_2'(t) = x_3(t)$$

$$x_3'(t) = x_4(t) + t$$

$$x_4'(t) = x_5(t)$$

$$x_5'(t) = \frac{1}{5}(2x_4(t) - 2x_3(t) + 1)$$

$$x_1(0) = x_2(0) = x_3(0) = 4, \quad x_4(0) = x_5(0) = 1$$