Zad. 4

$$\begin{cases} x' = x - y \\ y' = y - 4x \end{cases}$$

$$\begin{cases} \mathcal{D}[x] = [x] - [y] \\ \mathcal{D}[y] = [y] - 4[x] \quad (3) \end{cases} \Rightarrow [y] = [x] - \mathcal{D}[x] \Rightarrow \mathcal{D}[y] = \mathcal{D}[x] - \mathcal{D}^2[x]$$

(1) i (2) do (3)

$$\mathcal{D}[x] - \mathcal{D}^2[x] = [x] - \mathcal{D}[x] - 4[x]$$

$$\mathcal{D}^2[x] - 2\mathcal{D}[x] - 3[x] = 0$$

$$\Delta = 4 + 12 = 16$$

$$(\mathcal{D}[x] - 3)(\mathcal{D}[x] + 1) = 0$$

$$x(t) = C_1 e^{3t} + C_2 e^{-1t}$$

$$[y] = [x] - \mathcal{D}[x] \quad (a)$$

$$[x] = \frac{[y] - \mathcal{D}[y]}{4} \quad (b)$$

$$\mathcal{D}[x] = \frac{\mathcal{D}[y] - \mathcal{D}^2[y]}{4} \quad (c)$$

(b) i (c) do (a)

$$[y] = \frac{[y] - 2\mathcal{D}[y] + \mathcal{D}^2[y]}{4}$$

$$\mathcal{D}^2[y] - 2\mathcal{D}[y] - 3[y] = 0$$

analogicznie jak przy $x(t)$

$$y(t) = C_3 e^{3t} + C_4 e^{-1t}$$