Summary of Stability Classification

Asymptotically stable – All trajectories of its solutions converge to the critical point as $t \to \infty$. A critical point is asymptotically stable if all of A's eigenvalues are negative, or have negative real part for complex eigenvalues.

Unstable – All trajectories (or all but a few, in the case of a saddle point) start out at the critical point at $t \to -\infty$, then move away to infinitely distant out as $t \to \infty$. A critical point is unstable if at least one of A's eigenvalues is positive, or has positive real part for complex eigenvalues.

Stable (or **neutrally stable**) – Each trajectory move about the critical point within a finite range of distance. It never moves out to infinitely distant, nor (unlike in the case of asymptotically stable) does it ever go to the critical point. A critical point is stable if A's eigenvalues are purely imaginary.

In short, as t increases, if all (or almost all) trajectories

1. converge to the critical point $\rightarrow$ **asymptotically stable**,
2. move away from the critical point to infinitely far away $\rightarrow$ **unstable**,
3. stay in a fixed orbit within a finite (i.e., bounded) range of distance away from the critical point $\rightarrow$ **stable** (or **neutrally stable**).

(0,0) - disease-free equilibrium

$$J = \begin{bmatrix} \beta_H - (\delta_H + d_H) & 0 \\ 0 & \beta_C - (\delta_C + d_C) \end{bmatrix}$$

Aby p. był asymptotycznie stabilny wartości własne muszą być ujemne. Aby wartości własne macierzy 2x2 byłyy ujemne - ślad macierzy <0, wyznacznik >0.

$$\begin{cases} \beta_H - (\delta_H + d_H) + \beta_C - (\delta_C + d_C) < 0 & \text{(ślad macierzy)} \\ (\beta_H - (\delta_H + d_H)) \cdot (\beta_C - (\delta_C + d_C)) > 0 & \text{(wyznacznik)} \end{cases}$$

$$\beta_H - (\delta_H + d_H) = a$$
$$\beta_C - (\delta_C + d_C) = b$$

$$\begin{cases} a + b < 0 \\ a \cdot b > 0 \end{cases} \Rightarrow a > 0 \land b > 0 \lor \underline{a < 0 \land b < 0}$$

$$\beta_H - (\delta_H + d_H) < 0 \quad \land \quad \beta_C - (\delta_C + d_C) < 0$$

$$\frac{\beta_H}{\delta_H + d_H} < 1 \quad \Big/ \cdot \frac{\Lambda}{\delta_S N} \qquad \frac{\beta_C}{\delta_C + d_C} < 1 \quad \Big/ \cdot \frac{\Lambda}{\delta_S N}$$

$$\frac{\beta_H \Lambda}{(\delta_H + d_H)\delta_S N} < \frac{\Lambda}{\delta_S N} \qquad \frac{\beta_C \Lambda}{(\delta_C + d_C)\delta_S N} < \frac{\Lambda}{\delta_S N}$$

$$\underline{R_H < \frac{\Lambda}{\delta_S N}} \qquad \underline{R_C < \frac{\Lambda}{\delta_S N}}$$