possible to 0.3 m without exceeding 0.3 m, when the ship is resting on the surface of Glia-4. (The limit of 0.3 m is imposed by unloading clearance requirements.)

(c) The other adjustable component on the landing system is the linear shock-damper, which may be adjusted in increments of $\Delta b = 500$ N-sec/m, from a low value of 1000 N-sec/m to a high value of 10,000 N-sec/m. It is desirable to make b as small as possible because a large b produces large forces at impact. However, if b is too small, there is some danger that the probe will rebound after impact. To minimize the chance of this, find the smallest value of b such that the springs are always in compression during the oscillations after impact. Use a minimum impact velocity $V_L = 5$ m/sec downward. To find this value of b, you will need to use a software package to integrate the differential equation.

B Spread of Staph Infections in Hospitals—Part I

Courtesy of Joanna Pressley, Assistant Professor, and Professor Glenn Webb, Vanderbilt University

Methicillin-resistant *Staphylococcus aureus* (MRSA), commonly referred to as staph, is a bacterium that causes serious infections in humans and is resistant to treatment with the widely used antibiotic methicillin. MRSA has traditionally been a problem inside hospitals, where elderly patients or patients with compromised immune systems could more easily contract the bacteria and develop bloodstream infections. MRSA is implicated in a large percentage of hospital fatalities, causing more deaths per year than AIDS. Recently, a genetically different strain of MRSA has been found in the community at large. The new strain (CA-MRSA) is able to infect healthy and young people, which the traditional strain (HA-MRSA) rarely does. As CA-MRSA appears in the community, it is inevitably being spread into hospitals. Some studies suggest that CA-MRSA will overtake HA-MRSA in the hospital, which would increase the severity of the problem and likely cause more deaths per year.

To predict whether or not CA-MRSA will overtake HA-MRSA, a compartmental model has been developed by mathematicians in collaboration with medical professionals (see references [1], [2] on page 312). This model classifies all patients in the hospital into three groups:

- $H(t)$ = patients colonized with the traditional hospital strain, HA-MRSA.
- $C(t)$ = patients colonized with the community strain, CA-MRSA.
- $S(t)$ = susceptible patients, those not colonized with either strain.

The *parameters* of the model are

- β_C = the rate (per day) at which CA-MRSA is transmitted between patients.
- β_H = the rate (per day) at which HA-MRSA is transmitted between patients.
- δ_C = the rate (per day) at which patients who are colonized with CA-MRSA exit the hospital by death or discharge.
- δ_H = the rate (per day) at which patients who are colonized with HA-MRSA exit the hospital by death or discharge.
- δ_S = the rate (per day) at which susceptible patients exit the hospital by death or discharge.
- α_C = the rate (per day) at which patients who are colonized with CA-MRSA successfully undergo decolonization measures.
- α_H = the rate (per day) at which patients who are colonized with HA-MRSA successfully undergo decolonization measures.
- N = the total number of patients in the hospital.
- Λ = the rate (per day) at which patients enter the hospital.

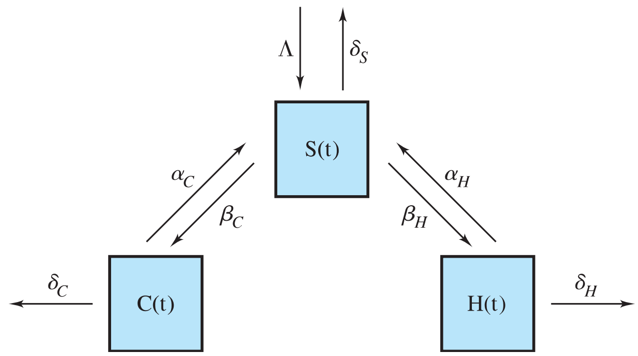

Figure 5.54 A diagram of how patients transit between the compartments

Patients move between compartments as they become colonized or decolonized (see Figure 5.54). This type of model is typically known as an SIS (susceptible-infected-susceptible) model, in which patients who become colonized can become susceptible again and colonized again (there is no immunity).

The transition between states is described by the following system of differential equations:

$$\frac{dS}{dt} = \underbrace{\Lambda}_{\text{entrance rate}} - \underbrace{\frac{\beta_H S(t)H(t)}{N}}_{\text{acquire HA-MRSA}} - \underbrace{\frac{\beta_C S(t)C(t)}{N}}_{\text{acquire CA-MRSA}}$$
$$+ \underbrace{\alpha_H H(t)}_{\text{HA-MRSA decolonized}} + \underbrace{\alpha_C C(t)}_{\text{CA-MRSA decolonized}} - \underbrace{\delta_S S(t)}_{\text{exit hospital}}$$
$$\frac{dH}{dt} = \underbrace{\frac{\beta_H S(t)H(t)}{N}}_{\text{from S}} - \underbrace{\alpha_H H(t)}_{\text{decolonized}} - \underbrace{\delta_H H(t)}_{\text{exit hospital}}$$
$$\frac{dC}{dt} = \underbrace{\frac{\beta_C S(t)C(t)}{N}}_{\text{from S}} - \underbrace{\alpha_C C(t)}_{\text{decolonized}} - \underbrace{\delta_C C(t).}_{\text{exit hospital}}$$

If we assume that the hospital is always full, we can conserve the system by letting $\Lambda = \delta_S S(t) + \delta_H H(t) + \delta_C C(t)$. In this case $S(t) + C(t) + H(t) = N$ for all t (assuming you start with a population of size N).

(a) Show that this assumption simplifies the above system of equations to

$$\textbf{(1)}\quad \begin{aligned} \frac{dH}{dt} &= (\beta_H/N)(N - C - H)H - (\delta_H + \alpha_H)H. \\ \frac{dC}{dt} &= (\beta_C/N)(N - C - H)C - (\delta_C + \alpha_C)C \end{aligned}$$

S is then determined by the equation $S(t) = N - H(t) - C(t)$.

Parameter values obtained from the Beth Israel Deaconess Medical Center are given in Table 5.4 on page 312. Plug these values into the model and then complete the following problems.

(b) Find the three equilibria (critical points) of the system (1).
(c) Using a computer, sketch the direction field for the system (1).
(d) Which trajectory configuration exists near each critical point (node, spiral, saddle, or center)? What do they represent in terms of how many patients are susceptible, colonized with HA-MRSA, and colonized with CA-MRSA over time?
(e) Examining the direction field, do you think CA-MRSA will overtake HA-MRSA in the hospital?

Further discussion of this model appears in Project E of Chapter 12.†

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†All references to Chapters 11–13 refer to the expanded text *Fundamentals of Differential Equations and Boundary Value Problems,* 6th ed.

TABLE 5.4 Parameter Values for the Transmission Dynamics of Community-Acquired and Hospital-Acquired Methicillin-Resistant *Staphylococcus aureus* Colonization (CA-MRSA and HA-MRSA)

Parameter	Symbol	Baseline Value
Total number of patients	**N**	**400**
Length of stay		
Susceptible	$1/\delta_S$	5 days
Colonized CA-MRSA	$1/\delta_C$	7 days
Colonized HA-MRSA	$1/\delta_H$	5 days
Transmission rate per susceptible patient to		
Colonized CA-MRSA per colonized CA-MRSA	β_C	0.45 per day
Colonized HA-MRSA per colonized HA-MRSA	β_H	0.4 per day
Decolonization rate per colonized patient per day per length of stay		
CA-MRSA	α_C	0.1 per day
HA-MRSA	α_H	0.1 per day

References

1. D'Agata, E. M. C., Webb, G. F., Pressley, J. 2010. Rapid emergence of co-colonization with community-acquired and hospital-acquired methicillin-resistant *Staphylococcus aureus* strains in the hospital setting. *Mathematical Modelling of Natural Phenomena* 5(3): 76–93.
2. Pressley, J., D'Agata, E. M. C., Webb, G. F. 2010. The effect of co-colonization with community-acquired and hospital-acquired methicillin-resistant *Staphylococcus aureus* strains on competitive exclusion. *Journal of Theoretical Biology* 265(3): 645–656.

C Things That Bob

Courtesy of Richard Bernatz, Department of Mathematics, Luther College

The motion of various-shaped objects that bob in a pool of water can be modeled by a second-order differential equation derived from Newton's second law of motion, $F = ma$. The forces acting on the object include the force due to gravity, a frictional force due to the motion of the object in the water, and a buoyant force based on **Archimedes' principle:** An object that is completely or partially submerged in a fluid is acted on by an upward (buoyant) force equal to the weight of the water it displaces.

(a) The first step is to write down the governing differential equation. The dependent variable is the depth z of the object's lowest point in the water. Take z to be negative downward so that $z = -1$ means 1 ft of the object has submerged. Let $V(z)$ be the submerged volume of the object, m be the mass of the object, ρ be the density of water (in pounds per cubic foot), g be the acceleration due to gravity, and γ_w be the coefficient of friction for water. Assuming that the frictional force is proportional to the vertical velocity of the object, write down the governing second-order ODE.

(b) For the time being, neglect the effect of friction and assume the object is a cube measuring L feet on a side. Write down the governing differential equation for this case. Next, designate $z = l$ to be the depth of submersion such that the buoyant force is equal and