$$\frac{dH}{dt} = \frac{(1-\eta)\beta_H}{N}(N-C-H)H - (\delta_H + \alpha_H)H$$

$$\frac{dC}{dt} = \frac{(1-\eta)\beta_C}{N}(N-C-H)C - (\delta_C + \alpha_C)C$$

$$\frac{dH}{dt} = 0$$

$$\frac{(1-\eta)\beta_H}{N}(N-C-H)H - (\delta_H + \alpha_H)H = 0$$

$$H = 0 \quad \vee \quad \frac{(1-\eta)\beta_H}{N}(N-C-H) - (\delta_H + \alpha_H) = 0$$

$$\frac{(1-\eta)\beta_H}{N}(N-C-H) = (\delta_H + \alpha_H) \quad \Big/ \cdot \frac{N}{(1-\eta)\beta_H}$$

$$N-C-H = \frac{N(\delta_H + \alpha_H)}{(1-\eta)\beta_H}$$

$$-H = \frac{N(\delta_H + \alpha_H)}{(1-\eta)\beta_H} - N + C$$

$$H = -\frac{N(\delta_H + \alpha_H)}{(1-\eta)\beta_H} + N - C$$

$$\frac{dC}{dt} = 0$$

$$\frac{(1-\eta)\beta_C}{N}(N-C-H)C - (\delta_C + \alpha_C)C = 0$$

$$C = 0 \quad \vee \quad \frac{(1-\eta)\beta_C}{N}(N-C-H) - (\delta_C + \alpha_C) = 0$$

$$\frac{(1-\eta)\beta_C}{N}(N-C-H) = (\delta_C + \alpha_C) \quad \Big/ \cdot \frac{N}{(1-\eta)\beta_C}$$

$$N-C-H = \frac{N(\delta_C + \alpha_C)}{(1-\eta)\beta_C}$$

$$C = -\frac{N(\delta_C + \alpha_C)}{(1-\eta)\beta_C} + N - H$$

$$(0,0)$$

$$\left(N - \frac{N(\delta_C + \alpha_C)}{(1-\eta)\beta_C}, 0\right)$$

$$\left(0, N - \frac{N(\delta_H + \alpha_H)}{(1-\eta)\beta_H}\right)$$

Linearyzacja

$$dH = F(C_0, H_0) \approx F_C(C_0, H_0)\,C + F_H(C_0, H_0)\,H$$

$$dC = G(C_0, H_0) \approx G_C(C_0, H_0)\,C + G_H(C_0, H_0)\,H$$

Pochodne cząstkowe

$$F_C(C_0, H_0) = \frac{-(1-\eta)\beta_H}{N} H_0$$

$$F_H(C_0, H_0) = \frac{(1-\eta)\beta_H}{N}(N - C_0 - 2H_0) - (\sigma_H + \alpha_H)$$

$$G_C(C_0, H_0) = \frac{(1-\eta)\beta_C}{N}(N - 2C_0 - H_0) - (\sigma_C + \alpha_C)$$

$$G_H(C_0, H_0) = -\frac{(1-\eta)\beta_C}{N} C_0$$

Po linearyzacji:

$$dH = \left[-\frac{(1-\eta)\beta_H}{N} H_0\right] C_0 + \left[\frac{(1-\eta)\beta_H}{N}(N - C_0 - 2H_0) - (\sigma_H + \alpha_H)\right] H_0$$

$$dC = \left[\frac{(1-\eta)\beta_C}{N}(N - 2C_0 - H_0) - (\sigma_C + \alpha_C)\right] C_0 + \left[-\frac{(1-\eta)\beta_C}{N} C_0\right] H_0$$

Macierz Jacobiego:

$$J = \begin{bmatrix} \frac{(1-\eta)\beta_H}{N}(N - C_0 - 2H_0) - (\delta_H + \alpha_H) & -\frac{(1-\eta)\beta_H}{N} H_0 \\ -\frac{(1-\eta)\beta_C}{N} C_0 & \frac{(1-\eta)\beta_C}{N}(N - 2C_0 - H_0) - (\delta_C + \alpha_C) \end{bmatrix}$$

Punkt (0,0)

$$J = \begin{bmatrix} (1-\eta)\beta_H - (\delta_H + \alpha_H) & 0 \\ 0 & (1-\eta)\beta_C - (\delta_C + \alpha_C) \end{bmatrix}$$

$$\det(J - \lambda I) = \det \begin{bmatrix} (1-\eta)\beta_H - (\delta_H + \alpha_H) - \lambda & 0 \\ 0 & (1-\eta)\beta_C - (\delta_C + \alpha_C) - \lambda \end{bmatrix}$$

Otrzymane wartości własne:

$\lambda_1 = (1-\eta)\beta_H - (\delta_H + \alpha_H)$

$\lambda_2 = (1-\eta)\beta_C - (\delta_C + \alpha_C)$

$$\lambda_1 < 0 \wedge \lambda_2 < 0$$

$$(1-\eta)\beta_H - (\delta_H + \alpha_H) < 0 \quad \wedge \quad (1-\eta)\beta_C - (\delta_C + \alpha_C) = 0$$

$$\frac{(1-\eta)\beta_H}{\delta_H + \alpha_H} < 1 \quad \wedge \quad \frac{(1-\eta)\beta_C}{\delta_C + \alpha_C} < 1$$

$$(1-\eta) \cdot R_H < \frac{\Lambda}{\delta_S N} \quad \wedge \quad (1-\eta) \cdot R_C < \frac{\Lambda}{\delta_S N}$$

Zeskanowane w CamScanner

Punkt $\left(N - \frac{N(\delta_C + \alpha_C)}{(1-\eta)\beta_C}, 0\right)$

$$J = \begin{bmatrix} \frac{(1-\eta)\beta_H}{N}\left[N - \left(N - \frac{N(\delta_C+\alpha_C)}{(1-\eta)\beta_C}\right)\right] - (\delta_H + \alpha_H) & 0 \\ \frac{-(1-\eta)\beta_C}{N}\left[N - \frac{N(\delta_C+\alpha_C)}{(1-\eta)\beta_C}\right] & \frac{(1-\eta)\beta_C}{N}\left[N - 2\left(N - \frac{N(\delta_C+\alpha_C)}{(1-\eta)\beta_C}\right)\right] - (\delta_C+\alpha_C) \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{\beta_H(\delta_C+\alpha_C)}{\beta_C} - (\delta_H+\alpha_H) & 0 \\ (\delta_C+\alpha_C) - (1-\eta)\beta_C & (\delta_C+\alpha_C) - (1-\eta)\beta_C \end{bmatrix}$$

$$\det(J - \lambda I) = \det\begin{bmatrix} \frac{\beta_H(\delta_C+\alpha_C)}{\beta_C} - (\delta_H+\alpha_H) - \lambda & 0 \\ (\delta_C+\alpha_C) - (1-\eta)\beta_C & (\delta_C+\alpha_C) - (1-\eta)\beta_C - \lambda \end{bmatrix}$$

Otrzymane wartości własne:

$$\lambda_1 = (\delta_C+\alpha_C) - (1-\eta)\beta_C$$

$$\lambda_2 = \frac{\beta_H(\delta_C+\alpha_C)}{\beta_C} - (\delta_H+\alpha_H)$$

$$\lambda_1 \wedge \lambda_2 < 0$$

$$(\delta_C+\alpha_C) - (1-\eta)\beta_C < 0 \quad \wedge \quad \frac{\beta_H(\delta_C+\alpha_C)}{\beta_C} - (\delta_H+\alpha_H) < 0$$

$$\frac{(\delta_C+\alpha_C)}{(1-\eta)\beta_C} < 1 \qquad \frac{\beta_H(\delta_C+\alpha_C)}{\beta_C(\delta_H+\alpha_H)} < 1$$

$$\frac{(1-\eta)\beta_C}{(\alpha_C+\delta_C)} > 1 \qquad \frac{\beta_H}{\delta_H+\alpha_H} < 1$$

$$(1-\eta)\cdot R_C > \frac{\Lambda}{\delta_S N} \qquad R_H < \frac{\Lambda}{\delta_S N}$$

$$R_H < \frac{\Lambda}{\delta_S N} < (1-\eta)R_C$$

Punkt $\left(0,\ N - \frac{N(\delta_H + \alpha_H)}{(1-\eta)\beta_H}\right)$

$$J = \begin{bmatrix} \frac{(1-\eta)\beta_H}{N}\left[N - 2\left(N - \frac{N(\delta_H+\alpha_H)}{(1-\eta)\beta_H}\right)\right] - (\delta_H + \alpha_H) & -\frac{(1-\eta)\beta_H}{N}\left(N - \frac{N(\delta_H+\alpha_H)}{(1-\eta)\beta_H}\right) \\ 0 & \frac{(1-\eta)\beta_C}{N}\left[N - \left(N - \frac{N(\delta_H+\alpha_H)}{(1-\eta)\beta_H}\right)\right] - (\delta_C + \alpha_C) \end{bmatrix} =$$

$$= \begin{bmatrix} (\delta_H+\alpha_H) - (1-\eta)\beta_H & (\delta_H+\alpha_H) - (1-\eta)\beta_H \\ 0 & \frac{\beta_C(\delta_H+\alpha_H)}{\beta_H} - (\delta_C+\alpha_C) \end{bmatrix}$$

$$\det(J - \lambda I) = \det \begin{bmatrix} (\delta_H+\alpha_H) - (1-\eta)\beta_H - \lambda & (\delta_H+\alpha_H) - (1-\eta)\beta_H \\ 0 & \frac{\beta_C(\delta_H+\alpha_H)}{\beta_H} - (\delta_C+\alpha_C) - \lambda \end{bmatrix}$$

otrzymane wartości własne

$\lambda_1 = (\delta_H + \alpha_H) - (1-\eta)\beta_H$

$\lambda_2 = \frac{\beta_C(\delta_H+\alpha_H)}{\beta_H} - (\delta_C + \alpha_C)$

$\lambda_1 \wedge \lambda_2 < 0$

$(\delta_H+\alpha_H) - (1-\eta)\beta_H < 0 \quad \wedge \quad \frac{\beta_C(\delta_H+\alpha_H)}{\beta_H} - (\delta_C+\alpha_C) < 0$

$\frac{(\delta_H+\alpha_H)}{(1-\eta)\beta_H} < 1 \qquad \frac{\beta_C(\delta_H+\alpha_H)}{\beta_H(\delta_C+\alpha_C)} < 1$

$\frac{(1-\eta)\beta_H}{\delta_H+\alpha_H} > 1 \qquad \frac{\beta_C}{\delta_C+\alpha_C} < 1$

$R_C < \frac{\Lambda}{\delta_s N}$

$(1-\eta)\cdot R_H > \frac{\Lambda}{\delta_s N}$

$R_C < \frac{\Lambda}{\delta_s N} < (1-\eta)R_H$